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ON A SIMULTANEOUS DECISION MODEL FOR MARKETING, PRODUCTION AND FINANCE†

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A previous study attempting to evaluate the benefits of coordination of marketing, production and finance decisions by Damon and Schramm is examined. It is noted that because of a flaw in the formulation of the marketing sector, the numerical results reported in this study are difficult to interpret. General issues in the design of experiments for evaluating functional coordination are also addressed.

Introduction

There appears to be a growing interest among management scientists in questions concerning coordination of intermediate range marketing and production decisions, and in recent years several different analytic approaches and models for evaluating the benefits of such coordination have been suggested (see for instance [1], [2], [4], [7], [8]). The most ambitious effort in this regard is undoubtedly that of Damon and Schramm [2], whose work is noteworthy both in terms of the scope of their model, which explicitly includes marketing, production and finance decisions, and in terms of their attempts to assess experimentally the likely magnitude of the economic benefits from coordination of such decisions. An exciting lead for future research in this field has been provided and subsequent modelling efforts will probably be judged to a significant degree against the standard provided by Damon and Schramm [2]. This is precisely why it seems appropriate to point out a few shortcomings of their formulation. While these do not diminish the overall intellectual and conceptual value of the work by Damon and Schramm, they are significant enough to warrant an explication which should help provide guidelines and perspectives for future work in the area.

1. Model Formulation

The main objective of the Damon and Schramm paper is assessing the economic benefits of coordinating marketing decisions with decisions in the production and finance sectors. Solutions with and without coordination are compared on the basis of the cash equivalent position of a small hypothetical firm at the end of a short-term planning horizon. Unfortunately, in the marketing sector model there are some serious structural problems, which cast considerable doubt on the accuracy of the reported results. These problems stem from the demand function.[1] [2, (13)]:

$$S_t = a_0 + a_1 S_{t-1} + a_2 A_t + a_3 / R_t, \tag{1}$$

where S_t, A_t and R_t represent the sales, advertising and price levels in period t and where the a_i's represent parameters to be estimated.

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[1] The Damon and Schramm notation is used as much as possible, but for notational convenience all time subscripts on model parameters are omitted.

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Damon and Schramm write, "There is considerable support in recent marketing litsrature for introducing the lagged demand in equation (13) to incorporate the effects of previous levels of advertising and price on current demand," [2, p. 165], but the majority of empirical studies of the lag hypothesis have in fact been concerned with current sales as a function of current and past advertising without any explicit incorporation of price variables in the lag part of the equation. For example, the estimation equation might be

$$S_t = a_0 + a_1 S_{t-1} + a_i f(A_t). \tag{2}$$

This relation can be motivated in several ways. For example, Simon writes: "If one makes some frequently reasonable assumptions, one can think of last period's sales as being the embodiment of the residual effect of all advertising up to the last period. And the same assumptions suggest that a certain proportion of last period's sales would carry over into this period. This is also consistent with the idea that last period's sales represent a stock of purchase habits, a large part of which would still be in effect in this period even if no advertising were done this period. Following on the above, one may use last period's sales as a variable in the regression, as a proxy for the effect of advertising in all prior periods" [6, p. 30].

Alternatively one can start with a relation of the form:

$$S_t = a_0 + b_0 f(A_t) + b_1 f(A_{t-1}) + \cdots b_i f(A_{t-i}) + \cdots . \tag{3}$$

Under suitable assumptions this relation can for estimation purposes be transformed into [2]. Kotler [3, Chapter 5] gives a good summary and discussion of some alternative formulations and transformations that can be used.

That the inclusion of price variables in these types of lag equations poses potentially troublesome questions is amply illustrated by (1). The problem here is that under certain conditions demands can remain positive no matter how high prices are, so arbitrarily large profits can be earned by posting exorbitant prices. This property of the model describing the marketing sector can be deduced by noting that the objective function for the marketing sector can be rewritten as

$$\sum_{t=1}^{T} \left\{ (R_t - d)\left(a_1^t S_0 + \sum_{i=0}^{t-1} a_0 a_1^i + a_2 a_1^i A_{t-i} + a_3 a_1^i / R_{t-i} \right) - \left(\alpha_{12} A_t + \alpha_{13} A_t^2 \right) \right\}, \tag{4}$$

where d is the constant marginal production cost assumed by the marketing sector.

By setting the price variables at arbitrarily large values in certain periods it is possible to make the objective function arbitrarily large. This occurs because demands remain finite so that the objective function will contain terms of the form $(R_t - d) \times$ constant, which of course approaches no finite limit as R_t goes toward infinity.

The crucial point is the sign of the expression $a_1^t S_0 + a_0 \sum_{i=0}^{t-1} a_1^i$ in the above objective function. If it is ever positive, then the objective function has no upper bound, since R_t can then grow without limit with demand remaining positive. Actually one need only consider the signs of the quantity $a_1 S_0 + a_0$. If it is positive, the objective function has no upper bound as R_1 grows without limit. Since the carryover parameter a_1 must be assumed nonnegative, $a_0 \leqslant 0$ is clearly a *necessary* condition for the existence of a finite optimum. *Sufficiency* conditions appear to be much more difficult to identify. For example if $s_0 = 0$, then letting R_1 grow without limit leads to negative demand if $a_0 \leqslant 0$, and consequently to an infinitely large loss. However, one can instead set R_1 quite low so that S_1 becomes very large (possibly incurring a first period loss) and then earn an infinite amount of profit by letting R_2 grow without limit. For example, using the parameter values given in [2], but assuming $S_0 = 0$, setting $R_1 = 2.68$ yields $S_1 = 1{,}006$, and $R_2 \to \infty$ then yields $S_2 =$ 1.85, and again the objective function has no upper bound.

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In essence, (1) needs to be replaced by some function S_t with the property $R_t \to \infty \Rightarrow S_t \to K$ where K is some finite number, though $K = 0$ would seem to be the most natural choice. One simple function which satisfies the above property would be

$$S_t = (a_0 + a_1 A'_t)/R_t, \quad \text{with } A'_t = a_2 A_t + a_3 A'_{t-1}. \tag{5}$$

The quantity A'_t can be thought of as the "current effective level of advertising" or what some authors refer to as "goodwill" [3], [5], [6]. To be sure, (5) does not incorporate any lagged price effects; however, there is at this point no firm empirical evidence on the nature and significance of lagged price effects. Furthermore, if such effects are indeed so significant that they *should* be incorporated into a dynamic model for price and advertising optimization, it is still not clear *how* they should be incorporated, but a few observations can be made. Specifications similar to (1) do not appear very suitable, for a low price in period $t-1$ effectively contributes positively toward demand in all subsequent periods, and there is no obvious and plausible explanation for this. In fact, one can reasonably assume that an increase in current demand generated by reducing the current price is at least partially attributable to an earlier placement of customers' orders. A price reduction would thus, *ceteris paribus*, reduce future demand. Generally speaking, a class of functions which appear to be quite flexible is $S_t = f_1(R_t) f_2(R_{t-1}, R_{t-2}\ldots, A_t, A_{t-1}\ldots)$ with the stipulations that

$$R_t \to \infty \Rightarrow f_1 \to 0$$

$$f_2 \to K; \quad \text{some finite number or bounded function}$$

Price and advertising now interact in determining demand, and if the current price is sufficiently high no amount of current advertising or previously accumulated goodwill can generate any demand.

2. Experimental Determination of Coordination Profits

2.1. *The Damon and Schramm Results*

The fundamental deficiency of the demand function (1) renders interpretation of the numerical results reported by Damon and Schramm rather difficult. As a concrete illustration of this, *one* globally optimal solution to the marketing sector model using the parameter values reported in [2] is shown in Figure 1. The quantity π_t is profit in period t.

t	1	2	3	4	5	6
A_t	0	0	0	0	0	0
R_t	∞	∞	∞	∞	d	d
S_t	41,700	12,617	3,850	914	$3500/d - 37$	$4555/d - 320$
π_t	∞	∞	∞	∞	0	0

FIGURE 1. An Optimal Solution to the Marketing Sector Model

The solution displayed in Figure 1 is but one of many solutions yielding an arbitrarily large profit. One might conjecture that the solutions presented in [2] are local optima. It is curious though that the possibly locally optimal prices are very close to the price in period 0. The number of variables and constraints is quite large, possibly large enough to seriously jeopardize the numerical accuracy of any penalty type of algorithm. This could well be the reason for the algorithm's not finding a direction with sharply increasing prices, so it seems that one cannot exclude the possibility that the reported solutions are not even local optima. This is unfortunate, but of course it does not invalidate the experimental design, which is quite sound.

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2.2. *Experimental Strategies in General*

To evaluate the benefits that are likely to arise from better coordination of marketing and production decisions, Damon and Schramm explicitly optimized the marketing system for a given set of marketing parameters and used the resulting demand distribution as an input when optimizing the production and finance sectors. Total cost under this sequential optimization procedure was compared to total cost when marketing and production decisions were optimized simultaneously. The essential aspect here is the comparison of the sum of two optima against a joint optimum. This is all very reasonable, and it is not at all clear how this experimentation strategy can be improved upon, but a number of questions remain. For example, is it at all possible to derive some measure of the *typicalness* of *specific* numerical illustrations? Damon and Schramm commendably caution the reader that "these specific numbers are dependent upon the rather arbitrary parameter values, but there was no attempt to bias the parameters toward enhancing the performance of the simultaneous model" [2, p. 170].

While this note offers no *definitive* resolutions of these kinds of ambiguities, what follows will hopefully shed some light on the issues involved.

In using the strategy of comparing the sum of two optima against a joint optimum a specific numerical experiment *could* indicate that coordination yields no benefits. Symbolically one has optimum (system 1 + system 2) $\geqslant$ optimum (system 1) + optimum (system 2).

Equality in the above relation will probably occur only under rare and peculiar circumstances, but a more precise characterization of these circumstances might nevertheless help identify improved experimentation strategies.

For the purpose of a simplified illustration of this point assume that marketing management is not quite so ignorant about production smoothing and seasonal considerations that they assume a time independent average direct production cost (d), but rather that they use a different cost (d_t) for each period t and consider the following marketing sector model where advertising is the only decision variable:

$$\max \sum_{t=1}^{T} (R_t - d_t)S_t - \alpha A_t^2 \quad \text{subject to } S_t = aA_t + \lambda S_{t-1}.$$

This model is essentially a much simplified version of the marketimg sector model used by Damon and Schramm. Prices are assumed fixed, the linear term in the advertising cost function has been omitted and there is no restriction on total advertising expenditure. The optimal advertising levels are

$$A_t = (a/2\alpha) \sum_{i=0}^{T-t} (R_{t+i} - d_{t+i})\lambda^i,$$

and the corresponding demand levels can be written as

$$S_t = \frac{a^2}{2\alpha} \sum_{i=1}^{T} (R_i - d_i)\lambda^{k_1} \sum_{j=0}^{k_2} \lambda^{2j}, \quad \text{with } k_1 = |t-i| \text{ and } k_2 = \min(t-1, i-1). \qquad (6)$$

The optimal advertising levels and the corresponding demand levels are thus linear functions of the costs d_t, and it is clear that the values chosen for d_t bear very directly on the magnitude of the incremental benefits from coordination. In particular there exist values of d_t for which the simultaneous and sequential solutions are *identical*, and in the above simple model these values are easily defined. In the sequential model, the optimal solution is a function of the sales levels determined by the marketing sector. Let $C^*(S)$ denote optimal cost for the production-finance sector as a function of sales. $C^*(S)$ can of course be optimized with respect to S; the resulting

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solution is denoted by $S^* = (S_1^* \cdots, S_T)$. By setting $S_t = S_t^*$ in (6) one obtains a system of simultaneous linear equations which can be solved for d. The solution to this equation system, call it d^*, gives the particular vector of unit costs for which coordination of the marketing and production-finance sectors will yield no economic benefits. In an experimental context the probability of randomly selecting $d = d^*$ is small, but the foregoing analysis shows that the *magnitude* of the benefits from coordination depend in a most crucial and direct manner on the specific values selected for the elements of the vector d. The same type of analysis can be carried out with respect to other subsets of parameters in the Damon and Schramm model, e.g., $\{a_{i,t}\}$ for $i = 0, 1, 2, 3$. While these observations do not lead to any definitive conclusions about proper experimental designs, the following appears to be an improvement over a random selection of d_t.

Initially let d_t represent the direct material and energy cost per unit of production, i.e., costs other than those attributable to production smoothing. Optimize the marketing sector and use the resulting demand distribution as an input when optimizing the production-finance sector. For each period calculate the average variable cost per unit according to the production-finance model solution. Add these unit costs to the previous costs d_t in the marketing sector, and *use these new unit costs* in comparing the performance of the sequential and the simultaneous models. This additional step in the comparative analysis is an attempt to correct for any unconscious bias toward coordination inherent in the selection of parameter values. Suppose, for example, that the initial choice of unit production costs yields a marketing sector solution which when used as an input to the production-finance sector leads to very high costs in this sector. If the original production costs were not adjusted, the comparative analysis would strongly favor the simultaneous solution. However, adding the unfavorable unit costs to the original cost estimates to some extent removes this initial bias by requiring the marketing sector to be partially cognizant of its impact in the production sector, even though a sequential solution procedure is used.

Assessing the typicalness of a particular numerical solution is a very difficult problem, which this note makes no claim to have resolved. It is hoped, however, that some stimulus toward this end has been provided.

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